Written Exam at the Department of Economics winter 2016-17

Micro III

Final Exam

Date: 9 Februrar 2017

(2-hour closed book exam)

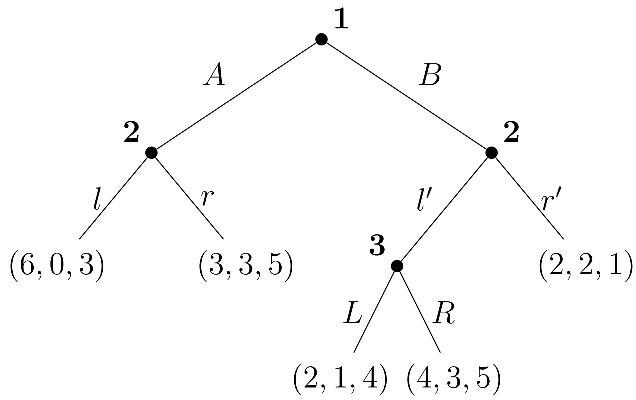
Please note that the language used in your exam paper must correspond to the language for which you registered during exam registration.

This exam question consists of 3 pages in total

NB: If you fall ill during the actual examination at Peter Bangsvej, you must contact an invigilator in order to be registered as having fallen ill. Then you submit a blank exam paper and leave the examination. When you arrive home, you must contact your GP and submit a medical report to the Faculty of Social Sciences no later than seven (7) days from the date of the exam.

PLEASE ANSWER ALL QUESTIONS.
PLEASE EXPLAIN YOUR ANSWERS.

1. Consider the extensive-form game given by the following game tree (the first payoff is that of player 1, the second payoff that of player 2, etc.):



   (a) Answer the following questions.
      i. Is this a game of perfect or imperfect information? How many proper subgames are there (excluding the game itself)? What are the strategy sets of the three players?
      ii. Find all (pure strategy) Subgame-perfect Nash Equilibria. Argue why you use the solution method you use.
      iii. Is the strategy profile (A, rr', L) a Nash Equilibrium? Discuss briefly (max. 3 sentences).

   (b) Consider again the game in (a), but suppose now that player 2 does not observe the move of player 1.
      i. Draw the resulting game tree.
      ii. Is this a game of perfect or imperfect information? How many proper subgames are there (excluding the game itself)? What are the strategy sets of the three players?
      iii. Find all (pure strategy) Subgame-perfect Nash Equilibria. Compare to the solution in (a).

2. Consider a *second-price sealed bid auction* with two bidders, who have valuations v_1 and v_2, respectively.

   Assume that the values are distributed independently uniformly with

$$v_i \sim u(0, 1).$$

   Thus, the values are **private**. Show that there is a symmetric Bayesian Nash Equilibrium where the players bid their valuation: $b_i(v_i) = v_i$ (recall that the auction format is second-price sealed bid).

3. Three entrepreneurs are considering starting a new tech company. They are free to form a company of any size between themselves. Entrepreneurs A and B are very experienced, with A being slightly more experienced than B, whereas entrepreneur C has no experience whatsoever. If entrepreneurs A and B work together in the company, the value of the company is 5 dollars (regardless of whether entrepreneur C joins the company). If entrepreneur A starts the company alone or with C, it is worth 2 dollars. If entrepreneur B starts the company alone it is worth 0 dollar, but if B starts it with C, it is worth 1 dollar. If entrepreneur C starts the company alone, it is worth 0 dollar.

   (a) Think of this situation as a coalitional game with transferable payoffs. Write down the value of each coalition.

   (b) Find the core of this game.

   (c) If all the entrepreneurs obtain a strictly positive payoff in the core explain why this is. If some entrepreneur receives a zero payoff in the core, explain why this is.

4. Consider the following version of Spence's education signaling model, where a firm is hiring a worker. Workers are characterized by their type θ, which measures their ability. There are two worker types: $\theta \in \{\theta_L, \theta_H\}$. Nature chooses the worker's type, with $\mathbb{P}(\theta = \theta_H) = p$ and $\mathbb{P}(\theta = \theta_L) = 1 - p$. Assume $p \in (0, 1)$. The worker observes his own type, but the firm does not.

   The worker can choose his level of education: $e \in \mathbb{R}^+$. The cost to him of acquiring this education is

$$c_\theta(e) = \frac{e}{\theta}.$$

   Education is observed by the firm, who then forms beliefs about the worker's type: $\mu(\theta|e)$. We assume that the marginal productivity of a worker is equal to his ability θ and that the firm is in competition such that it pays the marginal productivity: $w(e) = \mathbb{E}(\theta|e)$. Thus, the payoff to a worker conditional on his type and education is

$$u_\theta(e) = w(e) - c_\theta(e).$$

   Suppose for this exercise that $\theta_H = 4$ and $\theta_L = 2$.

   (a) In a separating equilibrium the low-ability worker chooses education level e_L and obtains wage $w_L = w(e_L)$. Is it possible that $e_L > 0$? Explain briefly (max. 3 sentences).

   (b) Find a separating pure strategy Perfect Bayesian Equilibrium where the two types choose education levels e_L and e_H, respectively, and the low ability type is indifferent between choosing e_L and e_H. Assume that off the equilibrium path, the firm assigns zero probability to the worker being type θ_H.

   (c) Find a pooling pure strategy Perfect Bayesian Equilibrium in which both types choose education level $\bar{e}$, and the low ability type is indifferent between choosing $e = 0$ and $e = \bar{e}$. Assume that off the equilibrium path, the firm assigns zero probability to the worker being type θ_H. Does the pooling equilibrium of (c) satisfy SR6? You can show this either graphically or algebraically.